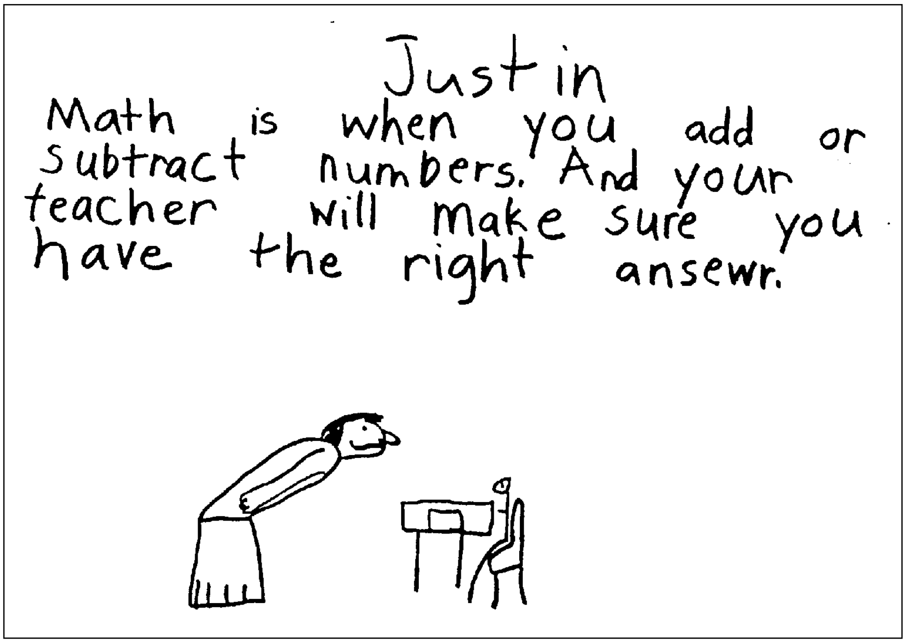
1 Talking, Writing, and Mathematical Thinking

Figure 1-1



When second-grade teacher Nancy Kerr asked her students at the beginning of the year to write about what mathematics was, Justin wrote and drew Figure 1-1.

Justin's brief commentary captures many of the dysfunctional beliefs that children come to associate with mathematics: only right answers count; teachers tell you how to get those right answers; working alone is the best way to improve one's competence.

This book focuses on ways to empower learners to think for themselves. It is a book about respecting children as sense-makers. We feel that there is no belief that can more radically change the teaching and learning of mathematics than this one. For if we view children as sense-makers, then we must also see them as story

Figure 1-1 originally appeared in "Ice Numbers and Beyond: Language Lessons for the Mathematics Classroom." *Language Arts* 74, no. 2 (February 1997): 108–15.

tellers, language creators, and problem-posers; we must value their background experiences and interests because we know these are important lenses for viewing their meaning-making efforts; we must value their stories because we know that stories are the way they frame their understanding of the world; we must value the many ways that they solve and pose problems because we see these as reflections of children's personal ways of thinking. In short, we must value their language, questions, descriptions, observations, and stories because these are windows into the process of how our students construct meaning. If we really view children as sense-makers, then we are required to step inside their shoes and view the world as they see it—through children's eyes.

As professional educators, we know that good teaching does not just happen through children's eyes alone. We must also take what we know about successful teaching and learning and use this knowledge to capitalize and extend children's current understandings of their world. As teachers of children, we must be close observers, keen listeners, and skillful questioners. However, we must also be reflective learners ourselves; we know that our own observations of children reflect our own beliefs about good learning. In this book we recognize sense making as the cornerstone of this belief system. We see mathematics and language as ways for learners to make sense of their world. Sense making in mathematics involves the strategic use of concepts, strategies, and skills. Concepts are the bedrock of mathematical thinking; they enable learners to view the world in a mathematical way (Steen, 1990; Paulos, 1988; Mills, O'Keefe, & Whitin, 1996): How long will it take me to complete this task (time)? What is the likelihood that I can get tickets for the concert (probability)? What is the cost of carpeting this room (area, money)? Strategies are "planful" ways to carry out the given task, such as estimating the cost of groceries in the basket; matching socks when doing the laundry; or counting the votes in a class election. Skills enable learners to obtain more specific answers, such as calculating the exact cost of those groceries or those square yards of carpet.

Writing and talking are ways that learners can make their mathematical thinking visible. Both writing and talking are tools for collaboration, discovery, and reflection. For instance, talking is fluid; it allows for a quick interchange of ideas; learners can modify, elaborate and generate ideas in a free-wheeling manner. Talking also allows for the quick brainstorming of many possible ideas, thereby giving the group many directions to consider. It is this "rough-draft" talk that allows peers and teachers a window into each other's thinking. As we talk with freshly fashioned ideas in our minds, we all witness the birth of still further ideas. Sharing partially formed ideas builds a willingness to live with the tentative and the provisional, an important dimension of a risk-taking stance (Lampert, 1990).

Writing shares many of the qualities of talking, but it has some unique characteristics of its own, such as creating a record of our thinking that we can analyze and reflect upon. Talking and writing enable learners to develop a personal voice. For example, after spending most of his fourth-grade year writing and talking about mathematical ideas, Jonathan sketched his interpretation of their benefits (Figure 1-2).

Jonathan showed that writing and talking are generative when he wrote, "You get more ideas." His repeated statement, "People get to know you," demonstrates his understanding that writing and talking are ways that learners can stamp their personal signatures on their mathematical thinking. Writing and talking are usually done with others in mind; children need the opportunity to share mathematical ideas in these ways so they can express what they know with a real audience.

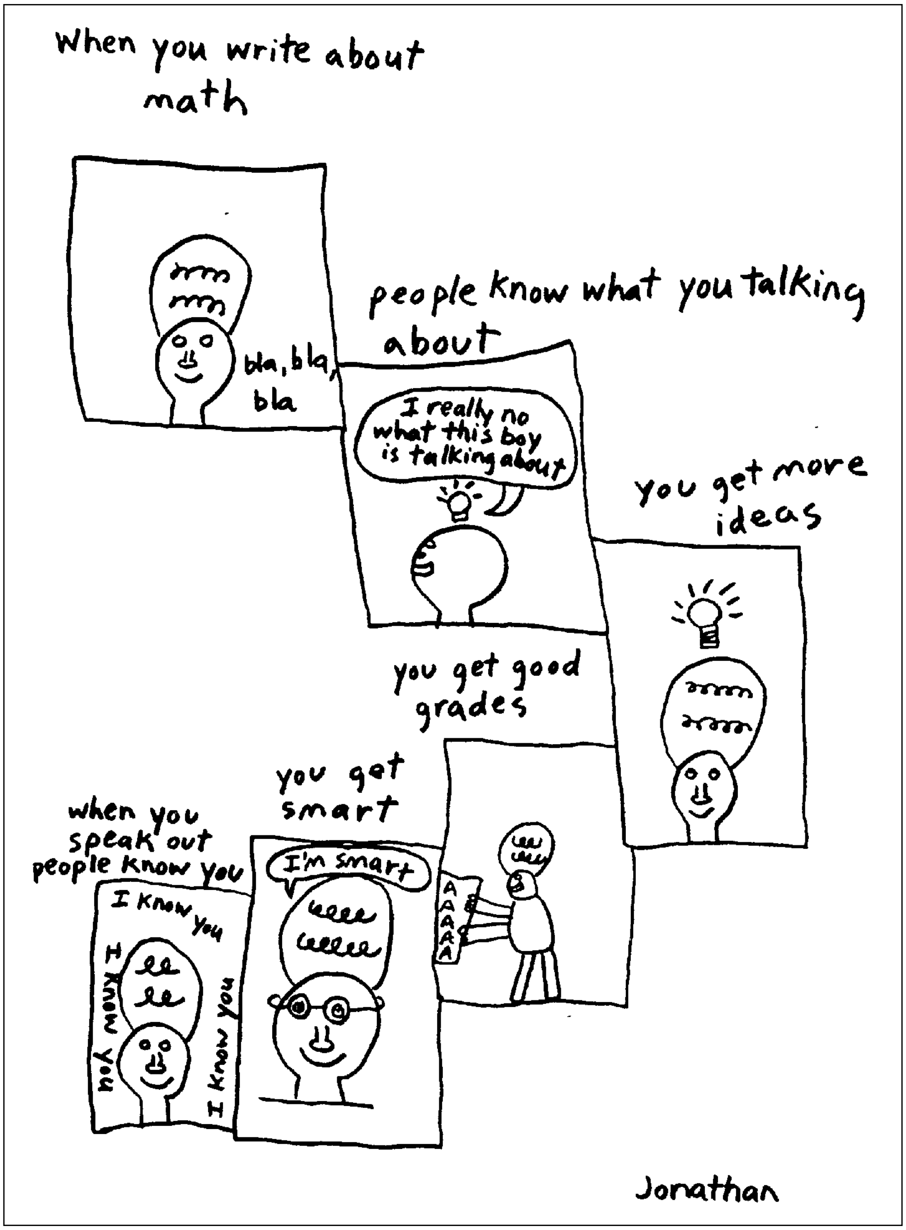
When children have regular invitations to write and talk about mathematics in open-ended ways, they soon recognize they can discover new ideas in the process. The following fourth-grade students wrote about this potential. Danielle wrote: [Conversations] helped me develop more ideas, and the more ideas, the more interested I got." Conversations with others draws learners deeper into an issue or problem. Lauren emphasized the same point when she wrote: "My picture shows that when one person raises their hand to say something everyone in the class raises their hand with a new idea" (Figure 1-3).

Meredith wrote: "Our ideas give others bigger ideas." Stephanie sketched the power of collaboration and discovery: "This sketch shows how when no one raises their hand everybody's brain light is off. This sketch shows how when someone raises their hand everyone's brain light is on" (Figure 1-4).

All of these children highlight one of Vygotsky's (1978) main ideas: talking does not merely reflect thought but it generates new thoughts and new ways to think. As members of a collaborative learning community the children are learning that together they can go further than any of them could go alone.

Writing is also a tool for discovery in mathematics. Lily wrote: "It (writing in mathematics) teaches me how to learn. When I write I get lots of ideas of what else I want to say." William reiterated this same idea: "You get more ideas. You get your imagination going." Jenny also reflected about the magic of discovering in writing: "When I write I get more ideas. See, I know what I'm going to write, but by the time I get to a part, I get a new idea." This potential of

Figure 1-2



writing and talking to discover new thoughts and ideas is one of the most important benefits for the teaching and learning of mathematics.

Sharing mathematical ideas through writing and talking builds a strong community of learners. Tiffany wrote of this mutual

Figure 1-3

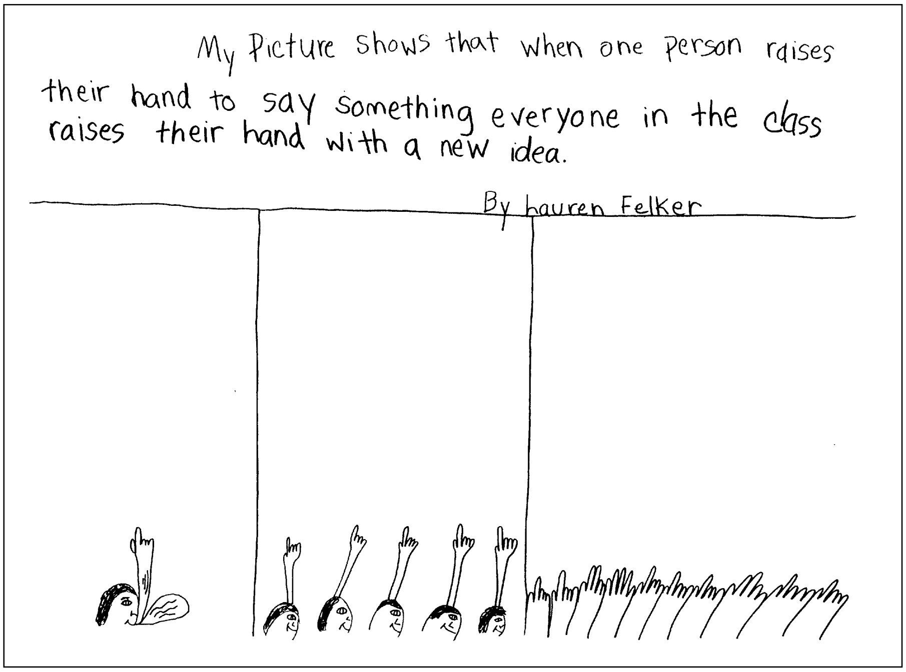


Figure 1-4

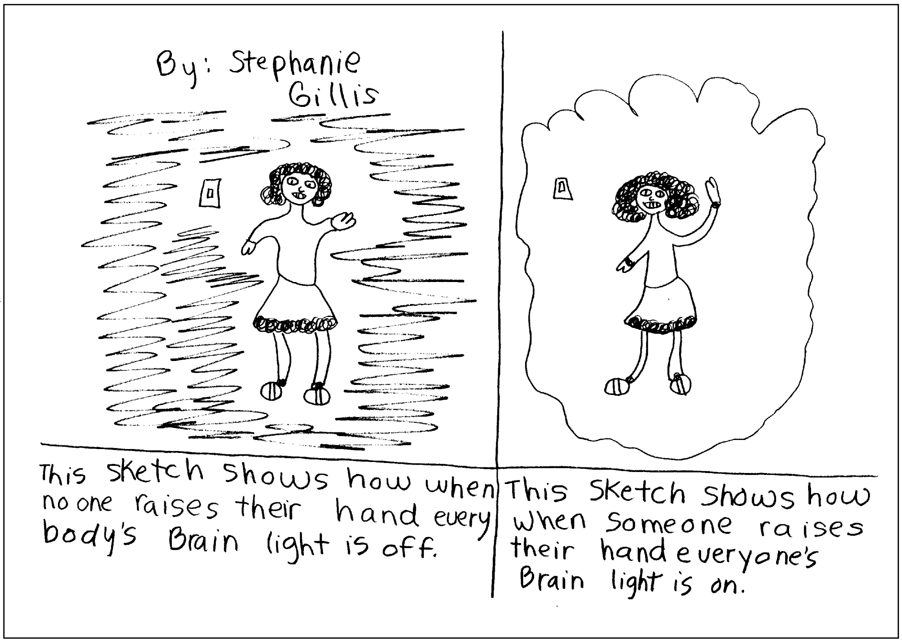
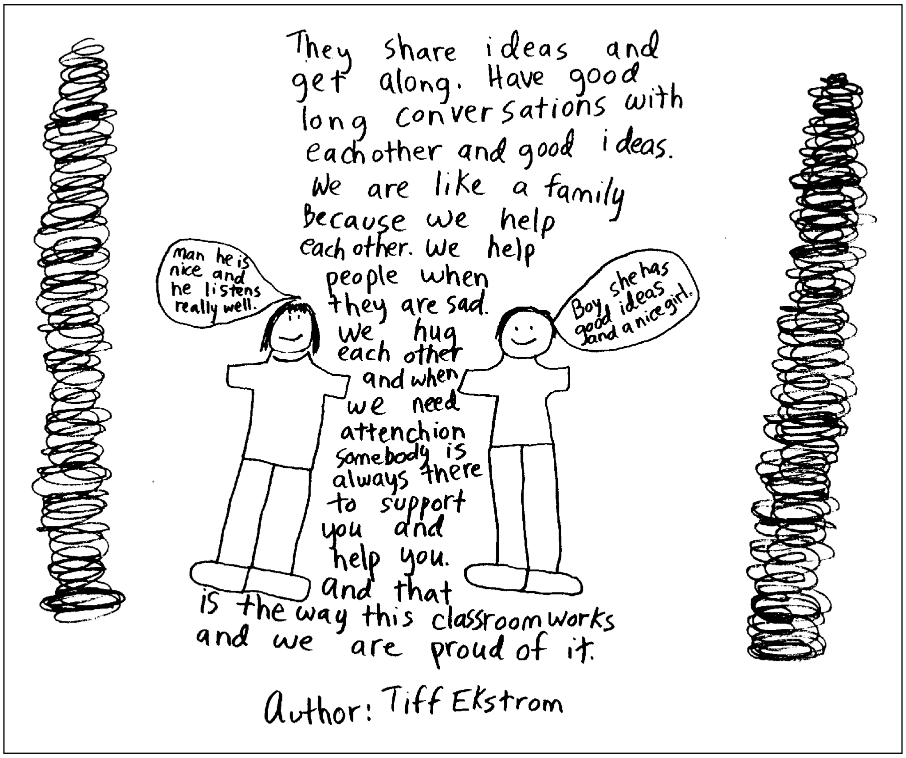


Figure 1-5



support by comparing it to the caring of a family: "They share ideas and get along. Have good long conversations with each other and good ideas. We are like a family because we help each other. . . . And that is the way that this class works, and we are proud of it" (Figure 1-5).

The risks that learners are willing to venture in mathematics are a reflection of the kind of community in which they live. Creating a community that supports the expression of mathematical ideas in many different forms is an important dimension of this risk-taking stance. In summary, writing and talking help to build a collaborative community that enables children to generate ideas, develop a personal voice, and reflect upon their current understandings.

The Teacher's Role in Building a Mathematical Community

National organizations in the field of language and mathematics have recognized the value of writing and talking as tools for understanding. The National Council of Teachers of Mathematics Standards recognizes this important role of language when it lists "learning to communicate mathematically" as one of the primary goals for all students. It advocates the development of "problem situations in which students have the opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking" (1989, p.6). It proposes that children have numerous opportunities to "realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics" (1989, p. 26). Likewise, the National Council of Teachers of English has been advocating the importance of using reading, writing, and talking for real purposes. In fact, both organizations emphasize a common set of beliefs about what constitutes good learning (Mills, O'Keefe & Whitin, 1996): (1) Learners are active constructors of their own knowledge; they are meaning-makers who are always making sense of problem situations by connecting them to what they already know; (2) Learners can represent their ideas through many forms of expression. Oral language, written language, mathematics, as well as drama, art, and music, are important channels for learners to express what they know about the world. (3) Learners construct knowledge in a social context. The way that learners construct what they know is influenced by the social situation in which they find themselves.

As natural as this potential is, it is not realized without the conscious decision-making of the teacher. How then do teachers build a community of mathematical thinkers who have opportunities to construct knowledge, express ideas, and share personal interpretations? The following list offers some strategies that we have developed for putting these beliefs into action:

1. *Highlight the process.* Children need to see that mathematics is more than a series of right-or-wrong answers. In order for them to value the process of mathematical thinking we have posed questions such as these:

"How did you solve that problem?"

"Did anyone solve it another way?"

"Tell us about what was going through your mind when you were working on this problem."

When we first invite children to explain their thinking at the beginning of the year, they sometimes look a bit hesitant, fearful that their answers are wrong. Many children have been conditioned by past classroom experiences to describe their thinking only when their answers were incorrect. By encouraging children to express their thinking in all circumstances, even when their answers might be the predicted response, we find that children have much to show us about their sense-making efforts.

We have also encouraged children to answer their own questions. Instead of all questions being directed to and answered by the teacher, we turn the questions back to the children: "Who can answer Deidre's question about this pattern?" In this way the conversation does not become a paired interchange between individual children and the teacher (Wood, 1998; Schwartz, 1996). It also demonstrates that individual questions are owned by the group and that everyone has a responsibility for each other's thinking and understanding.

2. *Recognize the thinking of others.* To build a community of learners who are willing to share their strategies and ideas with others, we must provide opportunities for people to recognize the thinking of others in many different ways. At the beginning of the year we teachers set the example by saying such things as "I appreciate the way Sara used a drawing to show how she traded ten units for one ten because it helped to show her thinking in another way," or "I appreciate how Jon used the metaphor of an overflowing cup to describe an improper fraction (5/3) because it gave me a picture in my mind of what happens when there is a quantity greater than one whole." It is important to accompany the appreciation with a reason so that learners know specifically the thinking behind the recognition. After we publicly express our appreciation we invite the children to do the same: "Who else would like to share an appreciation with Sara?" or "What else do people appreciate about the way Jon described his answer?" If the children have nothing else to contribute we usually add another appreciation ourselves before we ask other children to share.

We share other kinds of appreciations as well and invite the children to do likewise:

"Who would like to share an appreciation to someone who helped them during our work time today?"

"Who would like to share an appreciation to someone for asking a question that helped you grow as a learner?"

"Who would like to share an appreciation to someone who shared a comment, idea or suggestion that helped you understand some of this math in a new way?"

The focus of these appreciations is how others have helped us *grow*, not merely on what we liked in some general way. For instance, when Jonathan shared that his strategy for solving $10 - 7 = ?$ was to think of it as $7 + ? = 10$ we asked for specific appreciations. A response such as, "I liked how Jonathan solved it in a new way" is not specific enough. Our reply is, "What made it a new way for you?" A child then elaborates, "Well, he made the subtracting problem into an adding problem and that was easier for me to think about." Not only does this specificity help Jonathan know what part of his thinking was appreciated, but it also contributes to the class's understanding of the relationship between addition and subtraction.

As teachers we do not want to hide our intentions from children. We want children to recognize the potential of how a collaborative community nurtures individual growth (Borasi, et al., 1998). We believe that when learners share their thinking with the group we all benefit from hearing different explanations, viewing different modes of expression, and reconsidering alternative ways to interpret a problem. If we really believe in these benefits of collaborative learning, then we must invite appreciations that highlight this potential.

In addition to inviting public appreciations there are two other strategies we have used to recognize the thinking of others: (1) the traveling overhead transparency; and (2) the naming of strategies. We invite a child to take an overhead transparency home one night and record (with written narrative and pictures) how he or she solved one of the homework problems. The next day that child explains his or her thinking with the class. Knowing that they are going to share their thinking the following day makes children even more reflective about their problem-solving efforts. Sharing their work on an overhead lends an aura of importance to the event as well. The child in charge is also responsible for answering questions from anyone who had difficulty with the problem. When children finish their sharing we invite a few of them to offer appreciations as a way to conclude the session. The children enjoy the deserved recognition and see again the value we place in understanding each other's thinking.

The second way that we value children's thinking is to name a mathematical strategy in honor of the child who invented it. For instance, when Laura Jane solved 9 + 7 she explained, "I took one from the 7 and added it to the 9 to make a ten, and then I added ten and six and got 16. It's easier when I make a ten." The children tested out the strategy on other facts for 9 (9 + 4, 9 + 5, 9 + 6 and so on) and found it to be useful in all these cases as well. To honor her thinking we asked Laura Jane what she wanted to call this strategy and she said "the nines strategy." From that day forward we referred to that strategy as "Laura Jane's nines strategy" and added it to a master list of strategies that we had posted in the room. Throughout the year the children would use Laura Jane's name when referring to this strategy; this ritual of naming not only honored the individual child but emphasized the social origins of the class's mathematical ideas.

3. *Honor surprise.* It is important to cultivate a classroom atmosphere in which surprise is acknowledged and valued. At the beginning of the year some children seem reluctant to share surprising observations with the class. (Our hunch is that their past instructional history has taught them that if they are surprised then they were

either not listening carefully to the teacher or they did not understand the teacher's directions.) However, if we view learners as respected sense-makers and meaning-makers, then surprise is a very healthy sign; it means that children are trying to make sense of a given situation and explain certain unexpected results. For instance, Danny used some blocks to show that 7 was an odd number because he could not build two towers of equal height. He later proved that 17 was also odd and predicted that 27, 37, 47, and so on, would also be odd. When his teacher (David) asked him about 70 he predicted it also would be odd because it had a 7. However, when other classmates showed that they could divide 70 into two equal parts, Danny had to admit that 70 was indeed even—and he was surprised! He was puzzled that some numbers had 7s and were odd and yet other numbers (like 70) were even. This surprise led him to examine the numbers more closely and with help from his classmates he saw that the critical difference lay in the one's place. As teachers we can honor surprise by asking children regularly, "Did anything surprise you today as you worked?" We can then capitalize on the potential of surprise by asking further, "*Why* was that surprising to you?" and "How can we explain what happened?" Since surprise is the catalyst that causes learners to revise their current theories and explain the results in a new way, it is important for teachers to encourage children to make public their surprises. If learners expect surprise they'll find it. If they don't, they won't. When children are valued as respected detectives, they will view surprise as part of their daily work.

Teachers are contributing members of the learning community and they, too, must share their surprises with the class. Sometimes the strategies that children share are surprising to the teacher. When a student shared that her strategy for figuring out 12 – 5 was to add the 5 and the 2 (from the 12) to get 7, the teacher (David) and the class were all quite amazed. She explained that the strategy worked for other combinations as well: 13 – 5 = 5 + 3 = 8, and 14 – 5 = 4 + 5 = 9. David and the children quickly tested out this strategy on other problems and found that it only worked when subtracting 5 in the subtrahend. However, the surprise led us all to wonder: why does this strategy work? David needed unifix cubes to act out the problem for himself, and only then could he understand why the strategy worked: by renaming the 12 as 5 + 5 + 2 he could then subtract either one of the 5s and still have the other 5 left over, which he then could add to the 2 to make 7. It was because the 10 was composed of two 5s that the strategy worked. Here again surprise led us all down the road of unexpected, yet rewarding, mathematical understanding.

4. *Invite reflection.* We have had children keep math journals and

asked them to record their reflections about most investigations in this journal. We tape a set of questions on the inside front cover of each journal as suggestions for writing:

1) What do you notice?
2) What do you find interesting?
3) What patterns do you see?
4) What surprises you?
5) What do you predict? Why?
6) What do your findings make you wonder?
7) What does this remind you of?

The children are not required to respond to each question every day. In fact, some questions are sometimes more appropriate for one experience than another; mathematical experiences that are rich in patterns invite a lot of reflection on patterns and possible predictions about those patterns, while a series of child-generated story problems might invite more reflection on the intriguing nature of the problems themselves. However, having this wide range of questions available for each experience allows children the opportunity to respond in many different ways.

We teachers have intentionally posed open-ended questions so that children have the freedom to respond to any interesting feature of the problem. We did not want to narrow the range of responses but instead sought a list of questions that would make each journal unique and different. Past experiences showed that even the question, "What did you learn today?" was not an effective one for eliciting a diversity of responses. Children tended to write about what they did rather than reflect on the nature of the learning that occurred. Responses such as, "We used the base ten blocks today and learned how to do some trading" did not make public the children's thinking. The question, "Do you have any questions?" was also not helpful because children were reluctant to admit questions, especially in the area of mathematics. The word *wonder* seemed to better capture this same spirit. *Wonder* seemed to be a less threatening word; the children had little previous school association with this word, which conjured up a non-judgemental invitation to extend the experience in new ways.

We included the last question, "What does this remind you of?" because we found that children naturally described observations in metaphorical terms. For instance, Danny was circling the multiples of four and eight on the hundred square chart. He noticed that each multiple of eight was always circled twice and reasoned that multiples of four have the multiples of eight inside them. He then shared a personal analogy: "Four is like eight's little sister; she follows him wherever he goes." Danny had a younger sister and probably drew upon some personal experiences to construct this wonderful connection. (Other examples of metaphors will be shared throughout the book to illustrate the potential this question has for inviting interesting mathematical insights.)

The list of questions also reflects the values and beliefs that we hold as teachers. We want to encourage children to look closely, find interesting things, detect patterns, predict outcomes, pursue surprises, and pose wonders. The questions we ask and the language we use define who we are as a classroom community (Lampert, 1990). We have come to realize the importance of the verbs that we use in our classroom community to convey the intentions of our mathematical decisions. Some of the verbs that we use most often with children include: *explore, investigate, invent, discover, revise,* and *pretend.* They can be found in the following challenges and questions:

> What did you find when you *explored* that pattern?
> What is another way we can *investigate* this problem?
> *Invent* a strategy to solve that problem.
> How did you go about *discovering* this relationship?
> How can you *revise* your theory to include this new information?
> *Pretend* that the problem is slightly different and see what happens.

The language that we use to describe our intentions sets the parameters for what is possible. As teachers, we have come to pay closer attention to our language because we know it sets the tone for our classroom climate.

We have described some of our basic beliefs about good mathematical learning and have outlined some initial strategies for supporting this belief system in the classroom. Now we turn to two classroom scenarios that show in more detail how these beliefs and strategies look in operation.

Who Owns the Questions? Who Owns the Solution?

An unexpected classroom event one September provided us with an opportunity to look closely at how we respond to children's questions. We had spent several days solving subtraction problems with base ten blocks. (As will be discussed in depth in Chapter 3, the children used the terminology of *unit* for the ones, *longs* for tens, and *flats* for 100 - square centimeter pieces.) On this day Phyllis asked the children to build the number 350 with blocks on their desks. She then asked them to remove (or subtract) 145 from this collection and describe their thinking process in their journals. After a few minutes, Phyllis asked the students to share the strategies that they used to arrive at their answers. Megan raised her hand, a worried look clouding her face. Glancing around her table, she admitted, "I got 105, but everyone around me got 205."

"Tell us what you did to get 105," Phyllis suggested.

Megan answered, "I crossed everything out on the top number."

2 4 10	4 10
~~350~~	3~~50~~
– 145	–145
105	205
(Megan's Solution)	(Other Children's Solutions)

Phyllis was pleased that Megan had taken the risk to share her different answer with the class but she did not want Megan to focus solely on the solution to this one problem. Rather she wanted her to look more generally at the reasons for regrouping. Phyllis thought that having several other children discuss their reasoning might help Megan understand the process in a more general way. Yet, already several children looked uninterested. She doubted that the majority of the class would invest energy into trying to understand Megan's thinking, suspecting that these children considered Megan's problem to be a solitary one, and that they were neither part of the problem nor part of the solution. On the spur of the moment, Phyllis decided to involve everyone simultaneously in Megan's present dilemma. She asked Megan's permission to have everyone consider this issue further, and then turned to the class to continue the conversation.

"That's a confusing problem," Phyllis began. "In this class people have lots of different ways to solve problems. I want all of you to think hard about Megan's problem, and to write Megan a response in your journals. In your own way explain to Megan why you don't have to trade flats, or cross out the hundreds place, in this problem. Then Megan will call on several of the *teachers* in this room to hear their explanations." She copied Megan's written problem on the board (see Figure 1-6), and then asked the children to write Megan a response.

As it turned out, there were several children who had difficulty explaining the regrouping process to Megan. (Even having the right answer does not ensure understanding.) When it came time for Megan to call on some classmates, those who had had trouble writing a response listened for their own sake, as well as Megan's. Other children explained very clearly to Megan the difference between needing to regroup and not needing to regroup. These children learned from the experience, too, by mentally searching for a way to express their understanding. They were proud of their explanations and were eager to share with the group. By turning the problem back to the entire class, all the students had the opportunity to benefit from Megan's question. Involving everyone carried an implied message that all members of the class were committed to help each other. Having children assume this responsibility minimized the chances that children would criticize Megan for an inaccurate answer. Instead, the class shared a common goal of making sense of a confusing problem. Meanwhile, Megan's role had changed from being the person who made a mistake to being the leader of a discussion.

Sharing Understandings with a Classmate

These are some of the written responses that the children shared with Megan:

> Megan, you do not have to take away a flat because you have enough longs left to take away the four from the other four. (Stephanie)
>
> You do not have to take a flat because the longs do not need help! Sometimes you do, and sometimes you do not. (Jamie)
>
> Megan, you don't have to take away a flat because then it wouldn't be in the hundreds and the answer would be 11 and it wouldn't make sense. (Tony)
>
> [Tony's '11' refers to the tens place, which would be the difference of fifteen tens minus four tens. His response shows what would happen to the tens place if a hundred were broken down into tens and moved to the tens place.]
>
> Megan, you don't have to take away an extra flat because you borrowed a long, so you can get units. If it was a 150 to 345 you would have to take the first one away! (Jessica)

The children's explanations highlighted some interesting points for Megan to consider. Stephanie emphasized the importance of only trading when it was necessary to do so; Tony reasoned that trading a one hundred for ten tens would result in too many tens in that column; Jamie and Jessica explained that trading is contextual, and that a problem like 345 – 150 *would* require the exchanging of a 100 piece. As children offered their ideas, Megan listened closely. Part way through the discussion, she broke into a grin. She told her friends that she now understood the difference in problems that needed a great deal of regrouping and those that did not. It was helpful that she had heard so many interpretations of the problem. The conversation also benefited Phyllis in another way. The students' comments had served as a window for assessment. By reading the journals and listening to the children talk, we adults could assess each child's understanding and appreciate the personal strategies of each individual.

After the discussion, Tiff, who had also been confused initially, added a comment in her journal. Her complete entry read:

> Megan, you don't have to take away a flat because. I can't help you because I don't get it either. And so I can't answer you. I am really, really sorry. I wish I could. Well, now I can answer you because I listened to everybody. The longs do not need help because they have enough but if they don't have enough you need to trade a flat. Thank you for understanding.

Tiff clearly regretted that she could not explain the process to her classmate at first. Listening to the many explanations not only helped Tiff to understand the problem herself, but it gave her the opportunity to fulfill the responsibility that she felt for the welfare of others.

As teachers, we were also intrigued with Megan's willingness to share her anomaly with the class. We asked her privately about the entire experience the following day. Megan explained, "I had never seen a problem where you didn't cross out everything." (In drill and practice exercises, similar problems are usually grouped together for "mastery.") "When I first saw the problem, I said, 'Oh, this is easy.' But when I got 105 I looked around and other people got 205, and I was confused. I said to myself, 'What happened?'"

Phyllis remarked, "Sometimes people see that they don't have the same answer as others, but they keep it to themselves. Tell me about how you decided to ask." Megan replied, "In this class everyone is a teacher. If I asked a question, more people could help." This conversation with Megan helped us value her growth as a risk-taker who was more willing now to seek support from her peers. She also grew in confidence because she realized that voicing her problem helped her classmates.

In this story the children rallied to support Megan as she made sense of a confusing subtraction problem. However, as the next story will show, there is no guarantee in any classroom that children (or adults!) will refrain from criticizing others at all times. We could have assumed that once the children learned about respect and responsibility, they would consistently treat one another in those ways throughout the year (as they had treated Megan on that September day). However, both children and adults can forget this most important lesson about courtesy in the busy whirlwind of classroom life. Although we did talk about our community rules throughout the year, an incident the following January needed our special attention.

Sustaining a Respect for Sense Making

Over the course of the year, the children regularly created mathematical story problems. Phyllis would select ten to fifteen of these problems and duplicate them for homework practice. On this occasion, the students generated some division problems for homework. The next day the author of each problem called on another child to answer the problem and then explain how he or she solved it. Craig called on Gaye to solve his problem: "There were 184 stickers. Half the stickers have a bird on them. How many stickers have birds on them?" Gaye replied "142." Before Craig had a chance to ask Gaye how she obtained that answer, several children uttered a sarcastic "Huh?" Phyllis was upset by their tone and responded, "I didn't like those 'huh's.' That was an obvious put-down tone, and I don't tolerate put-downs in this classroom. The 'huh' sounded like, 'Boy, that's a stupid answer.' It sounded like, 'My answer is right, so I'm better.' I am angry about the 'huh.' Gaye's answer made sense to her, and we need to know what she was thinking so we can understand her answer."

Phyllis then asked Gaye to explain her reasoning. Gaye pointed to the numerals of 184 and said, "Half of four is two, and half of eight is four, and I just brought down the one." Phyllis realized the wonderful thinking that Gaye's strategy entailed; she had found half of 84 was 42 but did not know what to do with the remaining digit. Phyllis explained that the one represented "100" and asked Gaye what was half of 100. Gaye replied "50" and then together they added the 42 to the 50 and got the answer of 92. The children were quite impressed with this alternative way to solve the problem.

Reflecting upon the Climate of Our Classroom

Phyllis then asked the children to reflect upon what had just happened: "What did we learn from this experience?" They talked together about Gaye's feelings and about the benefits of appreciating each child's thinking. After reflecting more about the incident that evening, Phyllis decided to ask the children to write about the conversation in their journals. The children's comments show a sensitivity to the implications that this incident had for the classroom community. James wrote that the conversation was beneficial because, "Gaye might not have tried to help out with homework anymore." Joseph wrote, "We really didn't show friendship," and Stephanie showed a concern for the future: "If Gaye had another good idea she might be afraid that people would be rude about it like they were." Jessica captured the tone of the conversation in a direct way: "I don't think people should judge your answer before your thinking." Jamie wrote about the value of alternative strategies: "That conversation was important because we can get different ideas and strategies when someone has a different answer." Tiffany reiterated this same point: "It is very rude because they might have a good explanation that might help you with another problem."

Tiffany also was concerned about Gaye's feelings: "It would hurt her feelings and make her feel unimportant." Nick compared the exploratory nature of good conversations to the opportunity to investigate a new fort for the first time. He also concluded his piece by relating the class' discussions of civil rights to this present issue:

> If Gaye didn't share her thinking we might have not understood; we just like to know everything. It's like you are at a new fort; you don't know what's there, so what do you do? Explore, find trails, look around, find things. The main topic is to explore; that is the challenge, that is the adventure. You have to EXPLORE. You just have to find out what she is thinking. If she knows some kind of math problem, and we don't know [how she solved it], she [won't be able to] help us with her thinking. You can't judge the skin or the voice. The inside is important, and the brain, that is important.

This brief classroom scenario highlights some of the important dimensions of a classroom community that we have mentioned earlier: honor surprise, value children as sense-makers, encourage a variety of ways to solve problems, and develop a climate that respects all learners. Phyllis's response to Gaye, "Tell us how you solved this problem," valued her as a sense-maker. And indeed Gaye's strategy made good sense once she had the opportunity to explain it. Phyllis's supportive response also demonstrated to the children the importance of honoring surprise. As teachers we need to show how to respond to an unexpected answer. We need to be curious enough to say, "That's interesting, tell us some more," and not dismiss it as careless or nonsensical. We need to seek out surprise, examine it, and learn from it.

Conclusion

Writing and talking enable learners to make their mathematical thinking visible. It is through writing and talking that teachers obtain a window into their students' thinking. Both writing and talking are tools for discovery, enabling learners to make new connections as they engage in the process. The fluid nature of talk allows for the quick brainstorming of many ideas while the permanent quality of writing provides an important trail of our children's thinking.

Teachers play a key role in capitalizing on these benefits of writing and talking. Part of this role involves establishing norms of classroom life that recognize and appreciate the reasoning of others; highlight the process of mathematical thinking as children use concepts, strategies, and skills in strategic ways; honor surprise as a natural and legitimate part of the learning process; and invite reflection and self-evaluation as avenues for personal growth. In these next three chapters we will show in more detail the benefits of writing and talking about mathematical ideas and the key role that teachers play in nurturing this development.